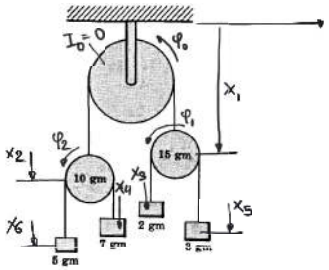
3.3 Beispiel dA03

Gegeben sei das skizzierte System aus Rollen und Massen.



(a) Bestimmen Sie sämtliche Bewegungsgleichungen des Systems.

(b) Bestimmen Sie die Beschleunigung der Masse 5m.

ZB: $x_1 + x_2 = l_1 = \text{const.}$, $\ddot{x}_1 + \ddot{x}_2 = 0$, $\delta x_1 + \delta x_2 = 0 \rightarrow \delta x_2 = -\delta x_1$

$x_3 + x_5 - 2x_1 = \text{const.}$, $\ddot{x}_3 + \ddot{x}_5 - 2\ddot{x}_1 = 0$, $\delta x_5 = 2\delta x_1 - \delta x_3$

$x_4 + x_6 - 2x_2 = \text{const.}$, $\ddot{x}_4 + \ddot{x}_6 - 2\ddot{x}_2 = 0$, $\delta x_4 = 2\delta x_2 - \delta x_6$

$\ddot{x}_4 + \ddot{x}_6 + 2\ddot{x}_1 = 0$, $\delta x_4 = -2\delta x_1 - \delta x_6$

Rollbed.: $x_3 - x_1 = r\varphi_1$, $\ddot{\varphi}_1 = \frac{\ddot{x}_3 - \ddot{x}_1}{r}$, $\delta\varphi_1 = \frac{\delta x_3 - \delta x_1}{r}$

$x_6 - x_2 = r\varphi_2$, $\ddot{\varphi}_2 = \frac{\ddot{x}_6 + \ddot{x}_1}{r}$, $\delta\varphi_2 = \frac{\delta x_6 + \delta x_1}{r}$

$I_1 = \frac{15}{2} m r^2$, $I_2 = 5 m r^2$

P.v.d'B:

$$\delta A^{(i)} + \delta A^{(a)} - \sum_{i=1}^{r} \left(m_i \underline{a}_i \cdot \delta \underline{r}_i + \underline{L}_i \cdot \delta \underline{\varphi}_i \right) = 0$$

$\delta A^{(i)} = 0$

$\delta A^{(a)} = 15mg\,\delta x_1 + 10mg\,\delta x_2 + 2mg\,\delta x_3 + 7mg\,\delta x_4 + 3mg\,\delta x_5 + 5mg\,\delta x_6$

$\sum_i m_i \underline{a}_i \cdot \delta \underline{r}_i = 15m\ddot{x}_1\delta x_1 + 10m\ddot{x}_2\delta x_2 + 2m\ddot{x}_3\delta x_3 + 7m(\ldots)$

$\sum_i \underline{L}_i \cdot \delta \underline{\varphi}_i = I_1 \ddot{\varphi}_1 \delta\varphi_1 + I_2 \ddot{\varphi}_2 \delta\varphi_2$

einsetzen:

$$\delta A^{(a)} = 15mg\,\delta x_1 - 10mg\,\delta x_1 + 2mg\,\delta x_3 - 14mg\,\delta x_1 - 7mg\,\delta x_6 + 6mg\,\delta x_1 - 3mg\,\delta x_3 + 5mg\,\delta x_6$$

$$\sum_i m_i \underline{a}_i \cdot \delta \underline{r}_i = 15m\ddot{x}_1\delta x_1 + 10m\ddot{x}_1\delta x_1 + 2m\ddot{x}_3\delta x_3 + 7m(-2\ddot{x}_1 - \ddot{x}_6)(-2\delta x_1 - \delta x_6) + 3m(2\ddot{x}_1 - \ddot{x}_3)(2\delta x_1 - \delta x_3) + 5m\ddot{x}_6\delta x_6$$

$$= (65m\ddot{x}_1 - 6m\ddot{x}_3 + 14m\ddot{x}_6)\delta x_1 + (-6m\ddot{x}_1 + 5m\ddot{x}_3)\delta x_3 + (14m\ddot{x}_1 + 12m\ddot{x}_6)\delta x_6$$

$$\sum_i \underline{L}_i \cdot \delta \underline{\varphi}_i = \frac{15}{2} m r^2 \frac{\ddot{x}_3 - \ddot{x}_1}{r} \frac{\delta x_3 - \delta x_1}{r} + 5mr^2 \frac{\ddot{x}_6 + \ddot{x}_1}{r} \frac{\delta x_6 + \delta x_1}{r}$$

$$= \left(\frac{25}{2}\ddot{x}_1 - \frac{15}{2}\ddot{x}_3 + 5\ddot{x}_6\right) m\,\delta x_1 + \frac{15}{2}(\ddot{x}_3 - \ddot{x}_1) m\,\delta x_3 + 5(\ddot{x}_6 + \ddot{x}_1) m\,\delta x_6$$

$$\Rightarrow -3mg\,\delta x_1 - mg\,\delta x_3 - 2mg\,\delta x_6 - (65\ddot{x}_1 - 6\ddot{x}_3 + 14\ddot{x}_6) m\,\delta x_1 - (-6\ddot{x}_1 + 5\ddot{x}_3) m\,\delta x_3 - (14\ddot{x}_1 + 12\ddot{x}_6) m\,\delta x_6 - \left(\frac{25}{2}\ddot{x}_1 - \frac{15}{2}\ddot{x}_3 + 5\ddot{x}_6\right) m\,\delta x_1 - \frac{15}{2}(\ddot{x}_3 - \ddot{x}_1) m\,\delta x_3 - 5(\ddot{x}_6 + \ddot{x}_1) m\,\delta x_6 = 0$$

Koeffizientenvergleich:

δx_1: $-3g - 65\ddot{x}_1 + 6\ddot{x}_3 - 14\ddot{x}_6 - \frac{25}{2}\ddot{x}_1 + \frac{15}{2}\ddot{x}_3 - 5\ddot{x}_6 = 0$

$$-3g - \frac{155}{2}\ddot{x}_1 + \frac{27}{2}\ddot{x}_3 - 19\ddot{x}_6 = 0 \qquad (1)$$

δx_3: $-g + 6\ddot{x}_1 - 5\ddot{x}_3 - \frac{15}{2}\ddot{x}_3 + \frac{15}{2}\ddot{x}_1 = 0$

$$-g + \frac{27}{2}\ddot{x}_1 - \frac{25}{2}\ddot{x}_3 = 0 \qquad (2)$$

δx_6: $-2g - 14\ddot{x}_1 - 12\ddot{x}_6 - 5\ddot{x}_6 - 5\ddot{x}_1 = 0$

$$-2g - 19\ddot{x}_1 - 17\ddot{x}_6 = 0 \qquad (3)$$

aus (2): $\ddot{x}_3 = -\frac{2}{25}g + \frac{27}{25}\ddot{x}_1$

aus (3): $\ddot{x}_6 = -\frac{2}{17}g - \frac{19}{17}\ddot{x}_1$

in (1): $\ddot{x}_1 = -\frac{196}{4423}g$

$\ddot{x}_3 = -\frac{14150}{110725}g$

$\ddot{x}_6 = -\frac{5134}{75293}g$